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Dissipative Formation of Hole-Like Excitation in Ion-Acoustic Plasma

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A viscous plasma is analyzed by reductive perturbation theory to model dissipative soliton formation. A nonlinear Schrödinger equation with complex coefficients is derived. Such an equation can be exactly solved by the technique due to Hirota. Three types of solution can be obtained under different physical conditions: solitary waves, ion-acoustic holes, and shocks. Even in the presence of a dissipative effect like viscosity, it is possible to obtain a solitary-wave-like excitation.

1. INTRODUCTION

Plasma physics is one of the most important domains for studying the formation and interaction of nonlinear waves (Shukla *et al.*, 1986). Of late, various situations have been studied and in some cases their relativistic generalizations have also been analyzed. Initial observations of ion-acoustic waves in experiments were made by Ikezi (1978). A detailed review of the present experimental status can be found in the excellent review of Longren (1983). The first derivation of the KdV equation in a plasma was done by Washimi and Taniuti (1966). Later some authors also performed higher-order calculations (Lai, 1979). Various modelings of physical situations have been done by incorporating the effects of two-temperature electrons (Tagare and Reddy, 1986), ion temperature (Nejoh, 1987), relativistic effects (Roy Chowdhury *et al.* 1988*a*; Das and Paul, 1985), and collisions (Kahawara, 1970) and Landau damping (Ott and Sudan, 1969; Roy Chowdhury *et al.*

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1986*b*). Others have also studied nonlinear wave formation in a magnetized plasma (Mio *et al.*, 1976). On the other hand, a variant of the original method developed by Washimi and Taniuti has been devised to consider envelope solitons, by considering a Fourier-like expansion of the disturbance (Ichikawa *et al.*, 1970). This type of methodology usually leads to a nonlinear Schrödinger (NLS) equation in place of the KdV equation.

If one critically reviews the different situations studied, it is apparent that there has not been extensive discussion of the effect of viscosity in a plasma medium. This may be due to the complex structure of the theoretical formulation. In a plasma, ions are chiefly responsible for the transport of momentum, and electrons for that of energy. Therefore, ions cause viscous effects and electrons cause thermal conduction. Thus, even in the absence of any static magnetic field, two natural units of length occur, one for the dissipation of Joule heat and the other for the viscous forces. These two lengths are not of the same order.

Under these circumstances we thought it to be worthwhile to investigate the influence of dissipation in particular situations in plasmas with the help of the flexible machinery of reductive perturbation theory. We use this theory to treat the viscous effect as a prototype of a dissipative force. We have deduced a new type of NLS equation with an extra linear term and complex coefficients. Such an equation is not known to be solvable through the inverse scattering Ablowitz (1978) method. However, it is interesting that one can adopt the methodology of Hirota (1981) to deduce multisoliton-like configurations for such an equation. Such an equation was previously studied from a purely mathematical point of view by Nozaki and Bekki (1984). Three distinct classes of waves may exist. One is the usual solitary type, another is a ion-acoustic hole, and the third is a shocklike object. Actually, dissipative effects such as viscosity are significant in magnetized plasma. But we have used it to simulate a dissipative force in a nonmagnetized system because the same type of NLS can be obtained in the magnetized situation, but with more labor.

2. FORMULATION

We consider a collisionless plasma consisting of isothermal electrons and cold ions. The effect of viscosity in the momentum transfer equation is also taken into account. To proceed further, we consider a fluid model for the plasma incorporating the effect of viscosity. The role of viscosity in plasmas has been discussed by Jackson (1975) and Chakraborty (1978).

These equations are written as

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}=-\frac{\partial \phi}{\partial x}+Q\frac{\partial^2 V}{\partial x^2}$$

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nV)=0 \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i$$

with

$$Q=\frac{1}{m}\left(\eta+\frac{4\zeta}{3}\right)$$

where η is the coefficient of bulk viscosity and ζ is the coefficient of shear viscosity. The quantities η an ζ are also known in the literature as the first and second coefficients of viscosity. Furthermore, if the ions are cold and the electrons are sufficiently warm, the thermal speed of the electrons will be much greater than the wave speed of the ion-acoustic wave. Hence, the effect of resonance particles may be small and the variation of the electron distribution function will be very small. Therefore, the effect of Landau damping due to electrons may be neglected. Hence we have assumed a fluid description of the plasma (Kakutani *et al.*, 1969)

We have taken the following physical situation:

$$V\to 0, \qquad n\to 1, \qquad \phi\to 0 \qquad \text{as} \quad x\to\infty$$

Stretched variables ξ and τ are defined via

$$\xi=\varepsilon(x-\lambda t) \qquad \tau=\varepsilon^2 t \tag{2}$$

Physical variables are expanded in a power series of the form

$$U=U^0+\sum_{\alpha=1}^{\infty}\varepsilon^\alpha\sum_{l=-\infty}^{\infty}U_l^{(\alpha)}(\xi,\tau)\exp\{il(kx-\omega t)\} \tag{3}$$

$$U=\begin{pmatrix} n \\ v \\ \phi \end{pmatrix} \tag{4}$$

Substituting these in the original set (1) and transforming to (ξ, τ), we obtain a set of equations for $U_l^{(\alpha)}$ by equating various powers of ε. Assuming the $l=0$ component of the first-order density perturbation $n_0^{(1)}$ is equal to zero, we obtain

$$U_l^{(1)}=0 \qquad \text{for} \quad l\neq 1$$

Furthermore, we have

$$\begin{aligned} V_1^{(1)} &= \frac{\omega}{k}\, n_1^{(1)} \\ \phi_1^{(1)} &= \frac{1}{1-k^2}\, n_1^{(1)} \end{aligned} \tag{5}$$

The compatibility condition is given as

$$(\omega + iQk^2)\,\frac{\omega}{k} = \frac{k}{1-k^2} \tag{6}$$

The group velocity is

$$-\frac{d\omega}{dk} = \frac{[2k/(1+k^2)^2] - 2iQ\omega k}{2\omega + iQk^2} \tag{7}$$

On the other hand, in second order of ε, we obtain

$$V_2^{(2)} = \frac{\omega k(1+4k^2)}{k[k^2-\omega(1+4k^2)\sigma]}\, Ln_1^{(1)} n_1^{(1)} - \frac{\omega}{k}\,(n_1^{(1)})^2 \tag{8}$$

where

$$\begin{aligned} L &= \frac{1}{2}\,\frac{k}{(1+4k)(1+k^2)^2} - \frac{1}{2}\,\frac{\omega^2}{k} - \frac{\omega}{k}\,\sigma \\ \sigma &= \omega + 2iQk^2 \end{aligned} \tag{9}$$

$$\begin{aligned} \phi_2^{(2)} &= \frac{k}{k^2-\omega(1+4k^2)\sigma}\, Ln_1^{(1)} n_1^{(1)} - M(n_1^{(1)})^2 \\ M &= \frac{1}{2}\,\frac{1}{(1+k^2)^2(1+4k^2)} \end{aligned} \tag{10}$$

$$\begin{aligned} n_2^{(2)} = \frac{k(1+4k^2)}{k^2-\omega(1+4k^2)\sigma} \bigg[& \frac{1}{2}\,\frac{k}{(1+4k^2)(1+k^2)} \\ & - \frac{1}{2}\,\frac{\omega^2}{k} - \frac{\omega}{k}\,\sigma \bigg] n_1^{(1)} n_1^{(1)} \end{aligned} \tag{11}$$

while the equations for $n_1^{(2)}$, $v_1^{(2)}$, and $\phi_1^{(2)}$ are

$$v_1^{(2)}=\frac{\omega}{k}n_1^{(2)}+i\left(\frac{\omega}{k^2}-\frac{\lambda}{k}\right)\frac{\partial n_1^{(1)}}{\partial\xi}$$
$$\phi_1^{(2)}=\frac{2ik}{(1+k^2)^2}\frac{\partial n_1^{(1)}}{\partial\xi}+\frac{1}{(1+k^2)}n_1^{(2)} \tag{12}$$

This time the compatibility yields

$$\lambda=\frac{2k/(1+k^2)^2-2iQ\omega k}{(2\omega+iQk^2)} \tag{13}$$

which is exactly the value given in equation (7). We now move to the set $n_0^{(2)}$, $v_0^{(2)}$, and $\phi_0^{(2)}$, for which we obtain

$$n_0^{(2)}=\frac{1}{1-\lambda^2}\left[\frac{1}{2(1+k^2)^2}-\frac{\omega^2}{2k^2}-\frac{\omega\lambda}{k}\right](n_1^{(1)}n_{-1}^{(1)}-C) \tag{14}$$

$$\phi_0^{(2)}=\frac{1}{1-\lambda^2}\left[\frac{1}{2(1+k^2)^2}-\frac{\omega^2}{2k^2}-\frac{\omega}{k}\lambda\right](n_1^{(1)}n_{-1}^{(1)}-C)$$
$$-\frac{1}{2(1+k^2)^2}(n_1^{(1)}n_{-1}^{(1)}-C) \tag{15}$$

$$v_0^{(2)}=\frac{1}{\lambda}\left\{\frac{1}{1-\lambda^2}\left[\frac{1}{2(k+k^2)^2}-\frac{\omega^2}{2k^2}-\frac{\omega}{k}\lambda\right]\right.$$
$$\left.+\frac{\omega^2}{4k^4}-\frac{1}{2(k+k^2)^2}\right\}(n^{(1)}n_{-1}^{(1)}-C) \tag{16}$$

The constant C is the boundary value of $|n_1^{(1)}|^2$ at $\xi=-\infty$.

We now equate the coefficients of ε^3, leading to the following set of equations:

$$\frac{\partial v_1^{(1)}}{\partial\tau}-i\omega v_1^{(3)}-\lambda\frac{\partial v_1^{(2)}}{\partial\xi}+ikv_0^{(2)}v_1^{(1)}+2ikv_2^{(2)}v_{-1}^{(1)}$$
$$=-\frac{\partial}{\partial\xi}\phi_1^{(2)}-ik\phi_1^{(3)}+Q\frac{\partial^2 v_1^{(1)}}{\partial\xi^2}+2ikQ\frac{\partial v_1^{(2)}}{\partial\xi}-Qk^2v_1^{(3)} \tag{17}$$

$$-i\omega n_1^{(3)}+ikv_1^{(3)}-\lambda\frac{\partial n_1^{(2)}}{\partial\xi}+\frac{\partial n_1^{(1)}}{\partial\tau}+ikn_0^{(2)}v_1^{(1)}$$
$$+\frac{\partial v_1^{(2)}}{\partial\xi}+2ikn_2^{(2)}v_{-1}^{(1)}+ikv_0^{(2)}n_1^{(1)}+2ikv_2^{(2)}n_{-1}^{(1)}=0 \tag{18}$$

$$\frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + 2ik \frac{\partial}{\partial \xi} \phi_1^{(2)} - k^2 \phi_1^{(3)}$$
$$= \phi_1^{(3)} + \frac{1}{2}\left[\phi_1^{(1)} \phi_0^{(2)} + 2\phi_2^{(2)} \phi_{-1}^{(1)} \right] - n_1^{(3)} \tag{19}$$

Therefore, elimination of other field variables in favor of $n_1^{(1)}$ leads to the equation

$$i \frac{\partial n_1^{(1)}}{\partial \tau} + \delta \frac{\partial^2 n_1^{(1)}}{\partial \xi^2} + \eta' |n_1^{(1)}|^2 n_1^{(1)} = \nu n_1^{(1)} \tag{20}$$

where the coefficients (δ, η', ν) are functions of physical quantities (ω, Q, k, λ). Equation (20) is a nonlinear Schrödinger-type equation with an inhomogeneous linear term on the right-hand side and complex coefficients. After some complicated analysis, equation (20) leads to the following result:

$$i \frac{\partial n_1^{(1)}}{\partial \tau} + (p_r + ip_i) \frac{\partial^2 n_1^{(1)}}{\partial \xi^2} + (q_r + iq_i) |n_1^{(1)}|^2 n_1^{(1)}$$
$$= i(AC_2 + BC_1) n_1^{(1)} + (AC_1 - BC_2) n_1^{(1)} \tag{21}$$

where the coefficients are complicated and lengthy functions of plasma parameters. We omit those expressions.

3. SOLUTION OF THE NONLINEAR SYSTEM

It is highly interesting to note that Nozaki and Bekki (1984) discussed a complex version of the generalized Ginzburg–Landau equation for obtaining exact solutions. Their motivation was a purely mathematical one, using the celebrated technique of Hirota (1981). We have written our equation (20) to conform with their notation. For the sake of completeness we briefly describe the basic methodology of the Hirota approach. In this approach the nonlinear field is sought as the ratio of two functions G/F and the usual differentiation is transformed to a "bilinear operator" D_x or D_t via the formal identity

$$D_x(f \cdot g) = \left(\frac{\partial}{\partial x'} - \frac{\partial}{\partial x} \right) f(x', t') \cdot g(x, t) \Big|_{x = x', t = t'}$$
$$= f_x g - f g_x \tag{22}$$

and various variations. In the particular case under consideration we set

$$n_1^{(1)} = e^{i(K\xi - \Omega\tau)} \frac{G(\xi, \tau)}{F^{1+i\Delta}(\xi, \tau)} \tag{23}$$

with K, Ω, and Δ real constants. Then equation (20) can be rewritten as

$$\frac{F}{qG}(\Omega - pK^2 - \Lambda + iD_{\Delta,\tau} + 2ikpD_{\Delta,\xi} + p \cdot D^2_{\Delta,\xi})(G \cdot F)$$

$$= \left(-\frac{p\hat{\Delta}}{q} D^2_{\Delta,\xi} + \frac{i\nu - \Lambda}{q}\right) \times (F \cdot F) - |G|^2 \tag{24}$$

Λ, Δ are to be determined. Here a modified bilinear operator $D_{\Delta,x}$ is defined through

$$D_{\Delta,\xi}(G \cdot F) = \left[\frac{\partial}{\partial \xi} - (1 + i\Delta)\frac{\partial}{\partial \xi'}\right] G(\xi)F(\xi') \qquad \text{at} \quad \xi = \xi'$$

$$\hat{\Delta} = \frac{(1 + i\Delta)(2 + i\Delta)}{2} \tag{25}$$

Usually equation (25) breaks up into two equations, one for $F \cdot F$ and the other for $F \cdot G$. One then seeks solutions of those in series form,

$$\begin{aligned} F &= 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \cdots \\ G &= g_1 + \varepsilon g_2 + \varepsilon^3 g_3 + \cdots \end{aligned} \tag{26}$$

where ε is a small parameter, and can be set equal to one after the calculation. One can keep terms in the series (26) up to any power of ε keeping consistency with (26). It has been observed that if $G = g_1$ and $F = 1 + f_1$, then we obtain a solution similar to the single soliton of the usual NLS equation, whence $G = g_1 + g_2$; $F = 1 + f_1 + f_2$ yields two soliton-like structures, and so on. However, because of the complex nature of the constants, these solutions are no longer pure solitons; their nature is completely different. In the following we discuss the various cases that may occur.

In our case, however, the coefficients are complex, and we must apply the Hirota approach directly as described above to find the solution.

Case a. Following Nozaki and Bekki (1984), we observe that if

$$\Delta = -\chi \pm (2 + \chi^2)^{1/2}$$

$$q_i\Lambda_r = q_r(\Lambda_i - \nu)$$

$$\chi = 1.5(p_r q_r + p_i q_i)(p_r q_i - p_i q_r)$$

and if we choose $\Lambda_i = v$ and $K = \Omega = 0$, we obtain a solitary-wave-like solution:

$$n_1^{(1)} = \frac{ge^{\Gamma}}{(1+e^{\Gamma+\Gamma^*})^{1+i\Delta}}, \qquad \Gamma = k\xi - \omega\tau \tag{27}$$

$$(k_r)^2 = (\mathrm{Re}\, k)^2 = \frac{3vA}{2q_i - p_i A(1+\Delta^2)}$$

$$k_i = k_r\Delta$$

$$|g|^2 = (4k_r^2)/A$$

$$\mathrm{Im}\, \omega = k_r^2[2p_i\Delta - p_r(1-\Delta^2)]$$

$$\mathrm{Re}\, \omega = 0$$

$$A = (p_r q_r + p_i q_i)/[|p|^2(2-\Delta^2)]$$

or a hole-type solution.

Case b. When K, $\Omega \neq 0$ we can have a solution

$$n_1^{(1)} = ge^{i(k\xi - \Omega\tau)} \frac{1-e^{2k\xi}}{(1+e^{2k\xi})^{(1+i\Delta)}} \tag{28}$$

whence

$$k^2 = -Av/(q_i + p_i\Delta^2 A)$$

$$|g|^2 = -k^2/A = (v + p_i k^2)/q_i$$

$$K = \Delta k; \qquad \Omega = p_r k^2 - g_r|g|^2$$

or a shock-type solution.

Case c. We can have a solution

$$n_1^{(1)} = g\frac{e^{i(K\xi - \Omega\tau)}\, e^{-k(\xi - S\tau)}}{1+e^{-k(\xi - S\tau)(1+i\Delta)}}$$

$$k^2 = -Av[q_i + A(\Delta p_i - 1.5p_r)^2/p_i]$$

$$K = 1.5kp/p_i \tag{29}$$

$$S = -3pi/p^2k$$

$$|g|^2 = k^2/A = [v + p_i(K + \Delta k^2)]/q_i$$

$$\Omega = p_r(K+\Delta k)^2 - q_r|g|^2 - S\Delta k$$

4. DISCUSSION

We have shown that nonlinear waves in a plasma having a viscous nature can be described by a new type of nonlinear Schrödinger equation. This NLS equation is exactly solved by using the Hirota technique. Three types of solutions can be obtained: (1) solitary-wave-like, (27); (2) hole type, (28); and (3) shock type, (29). The existence of each solution depends on the values of K, Δ, g, Ω, and S, which are defined by p_r, p_i, q_r, q_i, k, and v (p, q, k, and v are defined in terms of physical parameters k, ω, Q, and λ). In other words, under different physical conditions we obtain different types of excitation. In the presence of an impurity such as negative ions in the fluid model of a plasma, Tagare and Reddy (1986) have graphically shown a hole-like structure for up to a certain percentage of negative ions. However, to our knowledge no attempt has been made to establish such a hole-like excitation, equation (28), because of the presence of dissipative effects such as viscosity. This hole-like structure under certain physical conditions may also play a dominant role in the confinement of plasma particles. Again, another interesting feature to note here is that under another set of definite physical conditions, the usual soliton-like structure is possible.

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